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# Multivariate geostatistics to build a geological model of Bešeňová hydrogeothermal structure, Liptov basin, Slovakia

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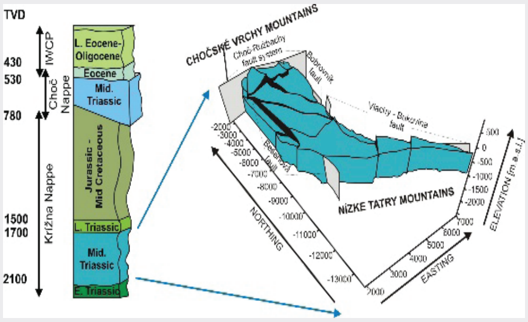
**Abstract:** Prior to the geothermal reservoir engineering and research in sense of modelling as dynamic flow, reservoir response, resource assessment or setting of production and injection sites, a 3D geological model is essential. The main emphasis in geological modelling of a subsurface structure is placed on correct spatial geometry and sequence of different chronostratigraphic or lithostratigraphic units for correct estimation and simulation of a spatial distribution of temperature and other studied reservoir parameters and variables used for geothermal reservoir assessment. The aim of the paper is to build 3D geological model of Bešeňová hydrogeothermal elevation structure using multivariate approach to preserve a hidden geometric correlation among seven geological units, without using any artificial correction like elimination of a negative thickness in some regions or total structure volume correction. The paper also compares the results of traditional sequential approaches based on univariate modelling of individual thicknesses for each zone or respective base horizons.

**Key words:** Bešeňová elevation, multivariate geostatistics, cross-variogram, cokriging

**Graphical abstract**



**Highlights**

- Construction of 3D geological models is essential for all aspects of geothermal reservoir engineering and geothermal research
- The presented paper deals with methodology and workflow of multivariate spatial modelling to derive an internal sequence of the base horizons of Bešeňová structure.

## Introduction

Maps and map making is an integral part of the work of a geologist. The geologists create maps of the subsurface geological structures and object that are hidden in the earth's crust. The final geological map represents a numerical model of a mapped phenomenon, such as thickness or depth of the studied structure (Yarus & Chamber, 2006).

Occurrence of thermal and mineral waters associated with the area of Bešeňová is well known for centuries, however, first systematic research dates to 1920's. Then, oil crisis prompted intense geophysical prospection on oil and gas resources during 1960 – 1970 in Slovakia, setting, meanwhile, a sound hints on geothermal potential in the country. This has also been a case of the Liptov Basin where the Bešeňová elevation is a part the basin.

Construction of 3D geological models is essential for all aspects of geothermal reservoir engineering and geothermal research. Geological models provide sound background for derived stationary and nonstationary geothermal modelling, dynamic hydrogeological flow models, interdisciplinary conceptual flow models, reservoir engineering models, such is a setting of production and injection sites, reservoir response modelling, discrete resource assessment, probabilistic simulations (e.g. the use of Monte Carlo based concept of geothermal reserves booking) etc.

The presented paper deals with methodology and workflow of spatial modelling of the zone thicknesses to derive an internal sequence of the base horizons of Bešeňová structure. A modelling approach is based on the multivariate geostatistical techniques that allow a mutual spatial modelling of more than one variable such as multivariate cokriging, collocated cokriging or kriging with external drift. Those multivariate techniques are primary used as an alternative way to a traditional sequential process of modelling based on a direct modelling of the internal horizons or surfaces, or modelling of the individual thicknesses of the zones or units per-partes. The paper also evaluates these two direct approaches to compare the results with using global approach for Bešeňová structure.

In general, geostatistical techniques of spatial modelling are model-based, therefore the paper also presents a complete modelling workflow including the used coregionalisation models composed of the direct variogram and respective cross-variograms. Due to the complexity of the variograms, only the simple omnidirectional models are presented.

### Site location and definition

The Bešeňová elevation forms elevated Mesozoic morphostructure running N-S direction in western part of the Liptov Basin (Fig. 1), defined to its surroundings along various tectonic systems and fault swarms (Maďar, 1997), i.e. the Liptovská Mara depression (E), the Ivachnová depression (W), Chočské vrchy Mts. (N) and Nízke Tatry Mts. (S) – see Fig. 2. As part of typical Tertiary intramountain depression of the Western Carpathians, its recent geological and tectonic arrangement into partial blocks of different uplift magnitude owes to combination of:

- pre-Paleogene relief breaking experiencing onset of first karstification (Činčura – Köhler, 1995);
- synsedimentary tectonic activity limiting Inner Western Carpathian Paleogene (IWCP) deposition rate (Gross et al., 1979) in Late Eocene – Oligocene;
- basin dissection and IWCP mass reduction during Neogene at N-S, NW-SE and SW-NE faults (Němec & Bartková, 1987);
- and finally rejuvenation of W-E faults during Late Neogene – Early Quaternary that inverted a relief to balance uplift tendencies of surrounding massifs (Jurewycz, 2005).

As a result, a first deep geothermal borehole targeting geothermal waters was installed in 1987 as ZGL-1 in Bešeňová (Fendek et al., 1988), with overall depth of 1 987 m, sampling thermal fluids at a wellhead temperature of 61.5 °C (Fig. 3). The Bešeňová elevation has been identified as a hydrogeothermal system, being a part of the Liptov Basin geothermal field, distinguishing several flow systems (circulation to accumulation dominated; e.g. Sorey et al., 1982) within Mid Triassic carbonates of the Choč and Krížna nappes. Recently, the Liptov Basin is also defined among geothermal water bodies of Slovakia. A total thermal potential of the structure has been estimated for 7.55 MWt (Fendek & Remšík, 2005; Remšík & Fendek, 2005; Remšík et al., 2005). Two new wells have later been installed, the shallow Fbe-1 with overall depth of 401 m (Vandrová et al., 2009) and the FGTB-1 well app.

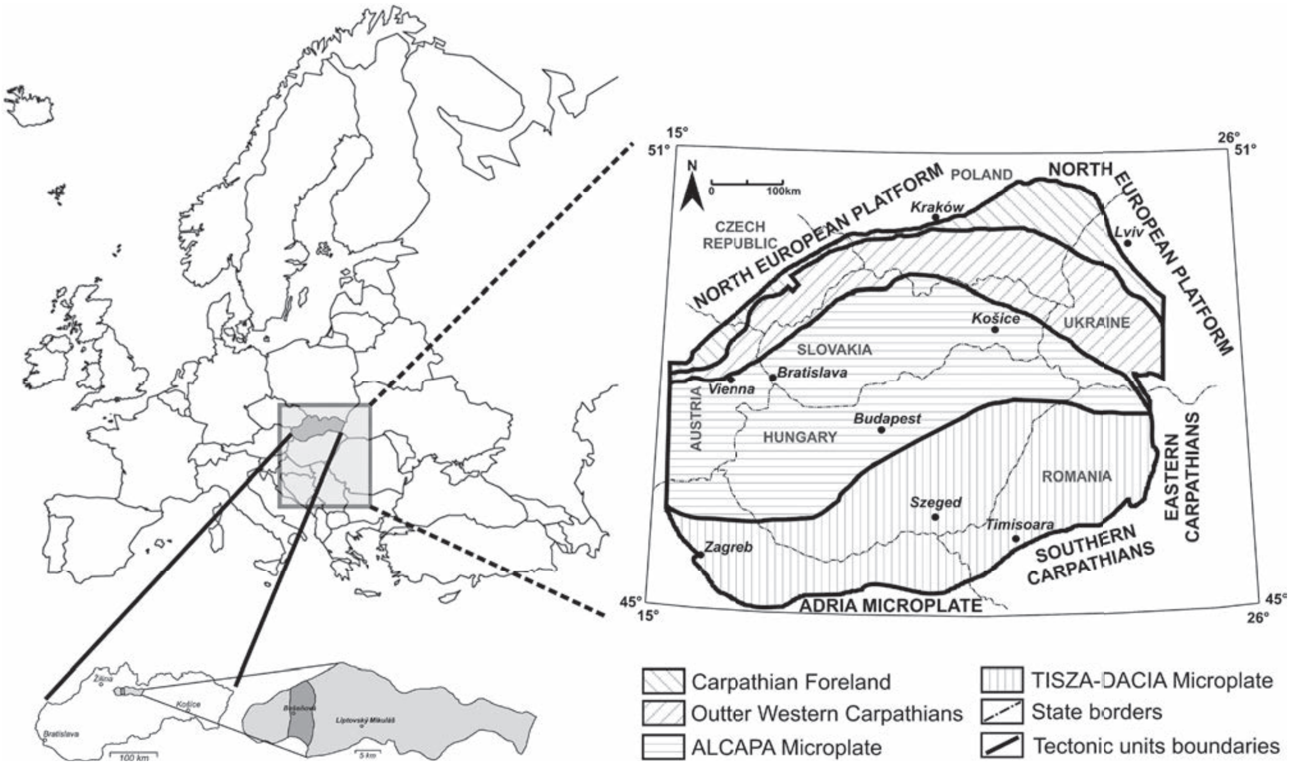


**Fig. 1.** Geographical and global-tectonic definition of the site. Modified after: Csontos & Vorös (2004), Tašárová et al. (2009).

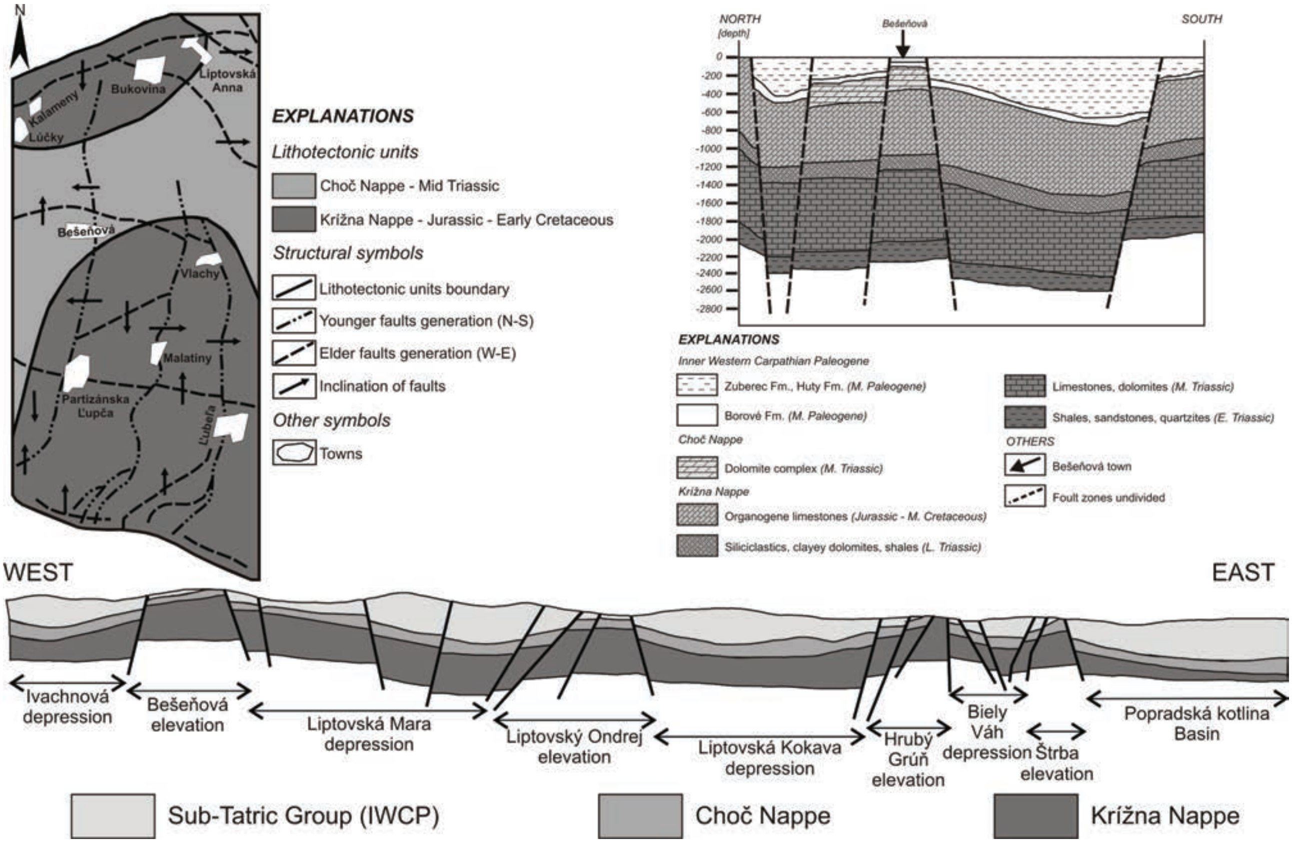


**Fig. 2.** Schematic cross section of the Bešeňová elevation (up) and the Liptov Basin geothermal field (down). Modified after: Gross et al. (1980).

1 833 m deep (Vandrová et al., 2011). The latest works carried magnetotelluric profiling at northern periphery of the structure (Fendek et al., 2017a), to solve hydraulic settings of its tectonic margin. An idealized (bottom – up) vertical profile corresponds then to (Gross et al., 1980; Remšík et al., 1998; Remšík et al., 2005; Fendek et al., 2017a, b):

- Devonian – Mid Carboniferous magmatites and metamorphites of the Tatricum Crystalline bedrock,
- para-autochthonous Early Triassic – Mid Cretaceous Tatricum Envelope Unit,
- allochtonous Krížna Nappe system,
- allochtonous Choč Nappe system forming several tectonic slags,
- Mid Eocene – Latest Oligocene succession of the Inner Western Carpathian Paleogene,

exclusively continental Quaternary sedimentary cover.

Given by geotectonic evolution of the entire basin, the idealized vertical profile is all but uniform through the Bešeňová elevation; assumed variable preservation of units in its partial blocks. Resolution of the model presented below does not involve the bedrock and the envelope unit, since not intercepted by any well installed in the system. Indeed, most of focus is given to position of what is recognized as two geothermal reservoirs (i.e. defined by Remšík et al., 1998; Fendek & Remšík, 2005).

The *shallow geothermal reservoir* forms a system composing of the Choč Nappe Mid – Late Triassic carbonates (dolomites prevail) in hydraulic connection with conglomerates, breccia and detritic to organogene limestones of the IWCP's Borové Formation. To the top, siliciclastics of the Huty (pelitic) and Zuberec Formation (flysch-like) form 10s to 100s m thick heat and hydraulic cap; rarely beneath the Biely Potok Fm. (mostly psammitic), all covered by Quaternary. Yet the reservoir is spatially reduced to a slag in central part of the elevation, not preserved to the north and south.

Unlike, the *deep geothermal reservoir* in Mid Triassic carbonates (dolomites and transient varieties privail) of the Krížna Nappe forms a solid body through the entire system. Above, clayey dolomites, clays and shales of the Late Triassic occur beneath beneath duplexed Jurassic – Mid Cretaceous succession including marlstones, claystones and their transient types; all repredenting upper insulator (Remšík et al., 1998; Vandrová et al., 2009, 2011). To the bottom, occurence of Early Triassic quartzites, shales and

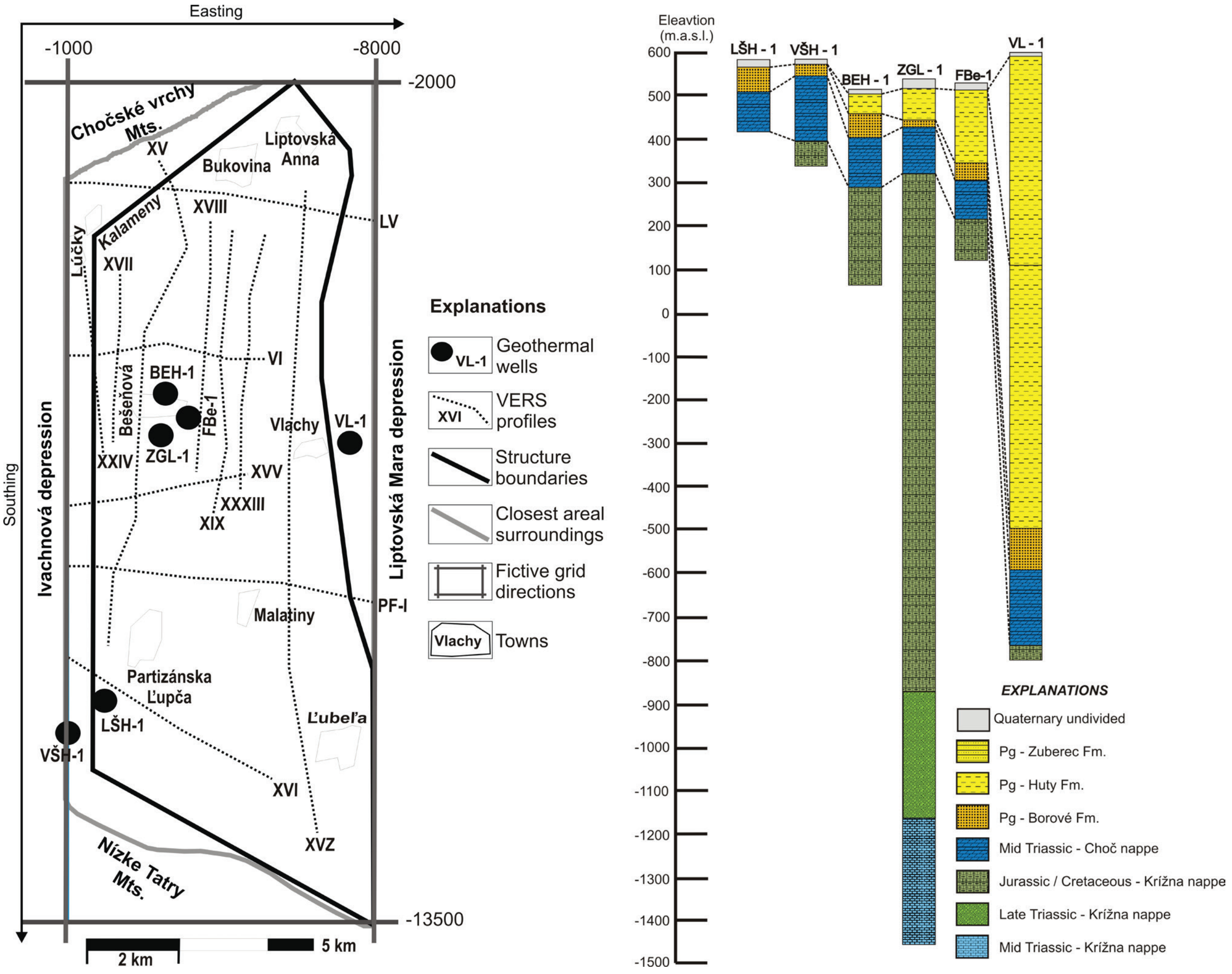


**Fig. 3.** Position and lithostratigraphical correlation of boreholes used in geological model construction.

quartzose sandstones is expected. Where the slag of the Choč Nappe is missing, the Krížna Nappe forms a pre-Paleogene basement.

## Methodology

The common feature of geometrical modelling in geology is a relation among the certain number of layers that constitute a formation or structure. It calls for a multivariate approach of spatial modelling to keep geometrical relations of sequence of the layers in the formation.

Geological model of the Bešeňová subsurface structure is based on geostatistical modelling. By definition (Matheron, 1962 in Journel & Huijbregts, 1978), *"Geostatistics is the application of the formalism of random function to the reconnaissance and estimation of natural phenomena"*. Geostatistics is based on theory of random function $Z(\mathbf{x})$ (function, which takes unknown shape during experiment) and its one realisation prof. Matheron called regionalised variable, $z(\mathbf{x})$, which is function of space coordinates **x** (Matheron, 1970, 1971). Geostatistical methods of estimation are model-based. That means they call for a spatial model of variability – **semivariogram** (shortly variogram). Variogram is a measure of one-half the mean square differences between the values of $z(\mathbf{x})$ and $z(\mathbf{x}+\mathbf{h})$ in two locations **x** and $\mathbf{x}+\mathbf{h}$ separated by a vector **h** (Olea, 1991):

$$2\gamma(\mathbf{h}) = \mathrm{E}\left[(Z(\mathbf{x}) - Z(\mathbf{x}+\mathbf{h}))^2\right] = \mathrm{Var}\left[Z(\mathbf{x}) - Z(\mathbf{x}+\mathbf{h})\right] \tag{1}$$

A **linear model of regionalisation**, composed of basic authorized structures of variability such as sprherical, exponential, Gaussian etc., is fitted to the experimental variogram (Goovaerts, 1997):

$$\gamma(\hbar) = \frac{1}{2N(\hbar)} \sum_{\alpha=1}^{N(\hbar)} \left( z(\mathbf{x}_\alpha) - z(\mathbf{x}_\alpha + \mathbf{h}) \right)^2 \qquad (2)$$

A natural extension of univariate modelling is multivariate approach, which is an extension of the concept of a single-variable regionalisation to several regionalised variables that are spatially inter-correlated (Dowd, 2004). Multivariate model of regionalization is called **linear model of coregionalisation**:

$$2\gamma_{ZS}(\mathbf{h}) = \mathrm{E}\left[ \left( Z(\mathbf{x}) - Z(\mathbf{x}+\mathbf{h}) \right) \cdot \left( S(\mathbf{x}) - S(\mathbf{x}+\mathbf{h}) \right) \right] \qquad (3)$$

Modelling of coregionalization of $N$ regionalised variables calls for inferring $N(N+1)/2$ direct and cross variogram models (Goovaerts, 1997).

A family of geostatistical estimation methods is called **kriging**. Kriging is estimation techniques, based on weighted linear combination of data and variogram model, which provides unbiased estimation with minimal variance of the errors (Isaaks & Srivastava, 1989). A major advantage of kriging over other interpolation methods is possibility to mutually krige more than one variable, so called **cokriging**. The estimation of unknow value in position $\mathbf{x}_\mathrm{o}$ for ordinary cokriging is given as a weighted linear combination of primary $Z$ and secondary $S$ data in the sample positions $\mathbf{x}_\alpha$:

$$z_\mathrm{o}^{*\mathrm{CK}} = \sum_{\alpha^Z=1}^{n_\mathrm{o}^Z} \omega_\alpha^Z z_\alpha + \sum_{\alpha^S=1}^{n_\mathrm{o}^S} \omega_\alpha^S s_\alpha \qquad (4)$$

Traditional methods of subsurface horizons modelling are based on a **sequential approach** of the estimation process, which consists in an individual processing of each surface (Sancevero et al., 2008). The result of this classical modelling approach is achieved through an estimation process of individual depth or thickness variables "per-partes". The next step consists in the individual processing of the obtained depth or thickness estimations from a reference surface to get the final stratigraphic surfaces of the layers. The disadvantage of this process is often observed crossing the modelled surfaces each other. That leads to the negative thicknesses in some regions what calls for artificial post-processing to eliminate these clearly unwanted results. In case of a separate thickness variable modelling often happens that the total thickness of the model mismatches the total thickness of the structure. In this case, a volume correction must be applied.

For this work, geostatistical modelling was chosen due to the possibility of a mutual modelling of stratigraphic surfaces. The aim of the Bešeňová structure modelling is application of an alternative way of modelling called **global approach** (Sancevero, et al., 2008). The global approach is based on multivariate approach of the spatial modelling of a sequence of the layers. Nevertheless there are many situations where it is not always possible to quantify adequately the correlation between the thicknesses of overlain beds, even though geologically and geometrically it must be present (Dowd, 1983). The global approach consists in modelling of the internal horizons of the structure where the layers, zones or units of a structure are modelled simultaneously under consideration of the inherent relationships among the layers. The used global approach in this study is based on the cumulative thicknesses of all the layers from a reference surfaces on the top of a formation to the base of the formation as follows:

$$\begin{cases} \mathrm{Z}_1^{\mathrm{cth}}(\mathbf{x}_\alpha) = \sum\limits_{i=1}^{1} \mathrm{Z}_i^{\mathrm{th}}(\mathbf{x}_\alpha) = \mathrm{Z}_1^{\mathrm{th}}(\mathbf{x}_\alpha), \\ \vdots \\ \mathrm{Z}_j^{\mathrm{cth}}(\mathbf{x}_\alpha) = \sum\limits_{i=1}^{j} \mathrm{Z}_i^{\mathrm{th}}(\mathbf{x}_\alpha), \\ \vdots \\ \mathrm{Z}_L^{\mathrm{cth}}(\mathbf{x}_\alpha) = \sum\limits_{i=1}^{L} \mathrm{Z}_i^{\mathrm{th}}(\mathbf{x}_\alpha) = \mathrm{F}^{\mathrm{th}}(\mathbf{x}_\alpha), \end{cases} \qquad (5)$$

where "th" denotes the thickness variables and "cth" denotes the cumulative thickness variables. The multivariate geostatistical methods are used for estimations of cumulative thicknesses and a total thickness of the formation. A detailed complex evaluation and comparison of the sequential and global approaches using simulated subsurface structure is given in Vizi & Benčoková (2015).

The total thickness of the formation serves as a secondary (auxiliary) variable, which controls the modelling results using a **collocated cokriging** method (Xu et al., 1992; Goovaerts, 1997; Wackernagel, 2003), and it is fully known between bounded top and base surfaces. The estimator for collocated cokriging is given as a weighted linear combination of primary $Z$ and secondary $S$ data with one extra value $s_\mathrm{o}$ of an auxiliary variable $S$, which is fully known in each unsampled location:

$$z_\mathrm{o}^{*\mathrm{CCK}} = \sum_{\alpha=1}^{n_o^Z} \left( \omega_\alpha^Z z_\alpha + \omega_\alpha^S s_\alpha \right) + \omega_\mathrm{o}^S s_\mathrm{o} \qquad (6)$$

Individual thicknesses of the respective layers are obtained from the total thickness of the formation and then the internal surfaces are calculated from a reference surface.

The main emphasis on the modelling of the horizons of the subsurface structures is given to the preservation of hidden geometric correlation between the geological units, which is based on two important assumptions:

1. The thickness of any zone within a specific formation must be equal to or greater than zero at each point of the studied domain. This means that the resulting *isochores map*, i.e. a map of vertical thickness, of the zone must be strictly positive (non-negative).
2. The sum of thicknesses of all zones within a particular formation must be equal to the total thickness of the formation.

## Data presentation

Resultant to application of geostatistic techniques is construction of complete geological model in a 3D scale. Involved zones are set due to combination of:

- stratigraphy (distinguishing Mesozoic and Paleogene sequences),
- tectonics (thin skinned Mesozoic nappe series and IWCP transgressive – regressive succession),
- hydrogeological regime (lithology-controlled permeability characteristics; recognizing *aquiferous horizons* – Krížna Nappe Mid Triassic carbonate complex, Choč Nappe Mid Triassic dolomite complex, IWCP Borové Formation; *aquitardous horizons* – zones of low permeability: Krížna Nappe Jurassic – Mid Cretaceous succession; *aquiclude horizons* – impermeable zones: Krížna Nappe Early Triassic, Krížna Nappe Late Triassic, IWCP Huty + Zuberec formations zone).

Presented model aims at the construction of a conceptual flow model of the Bešeňová elevation hydrogeothermal structure (Fričovský, 2014), matching its purpose in multiple topics (e.g. Fričovský et al., 2014, 2015, 2016). Although not all are presented in this paper due to a size limits and its primary scope, presented model is resultant to analysis and (re)interpretation of multidisciplinary achievable, mostly hard-copy data, such is:

- borehole technical reports with well profiles available upon time of model construction (i.e. Franko et al., 1979; Fendek et al., 1988; Vandrová et al., 2009, 2011);
- gravimetry survey maps (Zbořil et al., 1972; Szalaiová & Stránska, 1973; Szalaiová & Hančinová, 1974; Szalaiová et al., 2008);
- vertical electrical resistivity soundings and logs (Zbořil et al., 1972; Tkáčová, 1983; Szalaiová et al., 1993; Zembjak et al., 1986); and
- structural maps of pre-Paleogene basement, thickness and base maps of the Krížna Nappe and the Choč Nappe.

Lack of relevant reference nodes predefined need to fictive coordinate-system set-up applicable at defined structural margins, providing a regular pattern of 195 data points of 500 x 500 m, with a respect to geodetic systems at that coordinates increase with Easting and decrease with Southing. Hard copies were applied to a grid prior to accessible data interpretation regarding basal depth or thickness of defined intervals as a background for onward spatial interpolation.

For a model, seven horizons were distinguished referring to stratigraphy and hydrogeothermal relevance (from top to base):

1 – combined IWCP Huty Fm. and Zuberec Fm., (H-Z Fm.) including Quartenary,
2 – IWCP Borové Fm. (B Fm.),
3 – Choč Nappe Mid Triassic carbonates (Ch N.),
4 – Krížna Nappe Jurassic – Mid Cretaceous variegated shallow to deep marine carbonates (J-C),
5 – Krížna Nappe (Kr N.) Late Triassic siliciclastics and pelitic carbonates (LT),
6 – Kr N. Mid Triassic carbonates (MT),
7 – Kr N. Early Triassic siliciclastics (ET).

Thereafter, the complete thickness of the profile starts with the base horizon of the H-Z Fm. complex, including the quaternary cover (zone 1), running deepwards to the base of Kr N. Late Triassic formations (zone 7). Resultant cross-section represents a sum of IWCP + Ch N. + Kr N. Based on the inputs, the complete structure thickness increases in N – S axis to the E. in major, whereby the Bešeňová elevation is tectonically limited at the Vlachy – Ľubeľa fault zone to the Liptovská Mara depression hydrogeotermal structure. In minor to the west, correspondent to low drop at the Bešeňová fault, as a margin of the system to the neighbouring Ivachnová depression. Affine to expectations is a rapid drop in thickness to the south, where the structure limits to the Nízke Tatry Mts. along SW – NE and E – W fault swarms and to the north, nearby the Choč-Ružbachy fault zone, which limits the structure to the Chočské vrchy Mts.

Transition zone in the centre is resultant to markable drop in total Paleogene thickness, partially compensated with increase in thickness of the Mesozoic profile, resultant to presence of Ch N. Mid Triassic dolomite complex. In fact, thickness of the Borové Fm. (zone 2) is of low impact on Paleogene profile, as implied by low correlation between the zone and total IWCP thickness. Generally, total Paleogene thickness records dramatic decrease towards centre of the structure, where the Paleogene mass is limited at tectonic systems delimiting central horst.

Thickness of the Ch N. Mid Triassic dolomite complex (zone 3) expresses weak trending and correlation to either the total mass or mass of the Mesozoic sequence. However, more preservation of the mass is found to the E and W, most probably as a result of pre-Paleogene tectonic evolution. Most probably, territory along N – S axis was exposed to denudation, probably fairly elevated, whereas to the W and E, the region was submerged or at least of depressive character. Where the Ch N. is missing, area was most probably of rocky promontory massif character.

The zone of Kr N. Jurassic – Mid Cretaceous complex (zone 4) reveals zone of mass preservation below the Ch N. profile in production part of the system, in a contrast to regions of thickness lows at the northern and southern margin or along N-S axis. Explanation is found in pre-Paleogene geography again. Exposed regions to the north and south suffered from elevated character and mass reduction, intense enough to denude Ch N. profile prior reaching and karstifying the Jurassic – Mid Cretaceous sequence. Thickness characteristics for deeper parts of the Mesozoic sequence must not be explained or analysed in terms of pre-Paleogene or neotectonic evolution, as there is no hint on complete Jurassic – Mid Cretaceous sequence denudation, thus those had not underwent any subaerial exposure. Variation in thickness must be, thereafter, resultant to paleoevolution in the Kr N. hinterlands.

Thickness of the Kr N. Late Triassic profile (zone 5) increases towards W – E axis.

Below, the Kr N. Mid Triassic carbonate complex (zone 6) represents the bottom (deep) reservoir body, however, not recording any significant trend. Still, thickness highs locate towards SE margin, forming several local maxima regions, whereas lows concentrate at the NE periphery of the structure. Alike the Ch N. Mid Triassic zone, the horizon forms a solid body through the entire Bešeňová elevation structure.

The Kr N. Early Triassic horizon (zone 7) represents the deepest part of modelled profile with no real trend and correlation to overall thickness.

Hence, sequences of Tatricum Envelope Unit or the Tatricum Crystalline bedrock formations were not hit in any wells, either not sufficiently identified in gravimetry maps or geophysical soundings, any spatial analysis and attempt would lead to enormous uncertainties and a risk of mis-interpretation, therefore they are not included in the model. In a meantime, hard-copies were referenced for the structural maps of pre-Paleogene basement, thickness and base maps of the Krížna Nappe and the Choč Nappe.

Horizon 1 and 2 were from boreholes, VERS profiles and gravimetry maps. Base of horizon 3 was not hit in all wells; however, base and thickness maps with VERS profiles provided data relevant enough to define its spatial properties in grid nodes. Horizons 4 and 5 were identified in depth and thickness essentially from VERS profiles. For spatial analysis of horizon 6, thickness and base maps were used in nodes and later correlated with the ZGL-1 well and VERS. Highest uncertainties took place with horizon 7, roughly identified in some of VERS profiles.

Basic statistical parameters of the thickness variables for each zone described above are summarised in Tab. 1. We can see that minimal value of the H-Z Fm. thickness is zero due to occurrence of null thickness values of formation in the northern part of the structure. Zero thickness values of the Ch N. represent absence of the nape and they are delimitated and filtered out by the respective polygon.

**Tab. 1**

Basic statistical parameters of the thickness variables for each zones constitute studied Bešeňová structure.

| Variable | Zone | Min [m] | Max [m] | Mean [m] | SD* [m] |
|---|---|---|---|---|---|
| H-Z Fm. | 1 | 0.00 | 1 390.00 | 515.18 | 327.74 |
| B Fm. | 2 | 30.00 | 100.00 | 53.90 | 15.33 |
| Ch N. | 3 | 0.00<br>(100.00) | 300.00<br>(300.00) | 69.74<br>(197.10) | 103.56<br>(72.17) |
| J-C | 4 | 100.00 | 1 130.00 | 602.82 | 207.65 |
| LT | 5 | 150.00 | 220.00 | 194,36 | 21,70 |
| MT | 6 | 400.00 | 800.00 | 647.69 | 129.43 |
| ET | 7 | 100.00 | 200.00 | 149.10 | 28.24 |
| TT** | | 1 160.00 | 3 145.00 | 2 232.79 | 417.10 |

*SD – standard deviation
**TT – total thickness of Bešeňová structure
( ) – excluded zero values

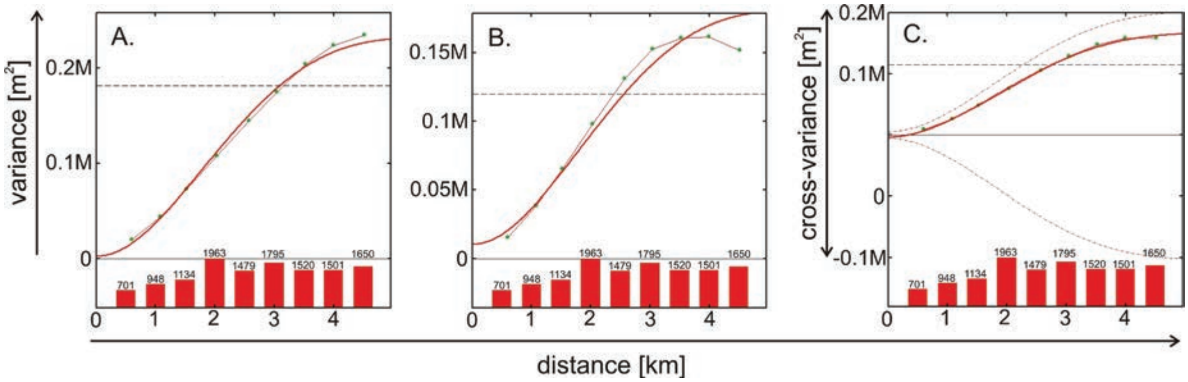


**Fig. 4.** Geostatistical model of spatial coregionalisation formed by direct variogram models of the total thickness of the Bešeňová structure (A.) and the Paleogene thickness (B.) and their cross-variogram (C.). The model is partialy used in Step 1 to get the total thickness map and in Step 2 to get Paleogene and Mesozoic thickness maps.

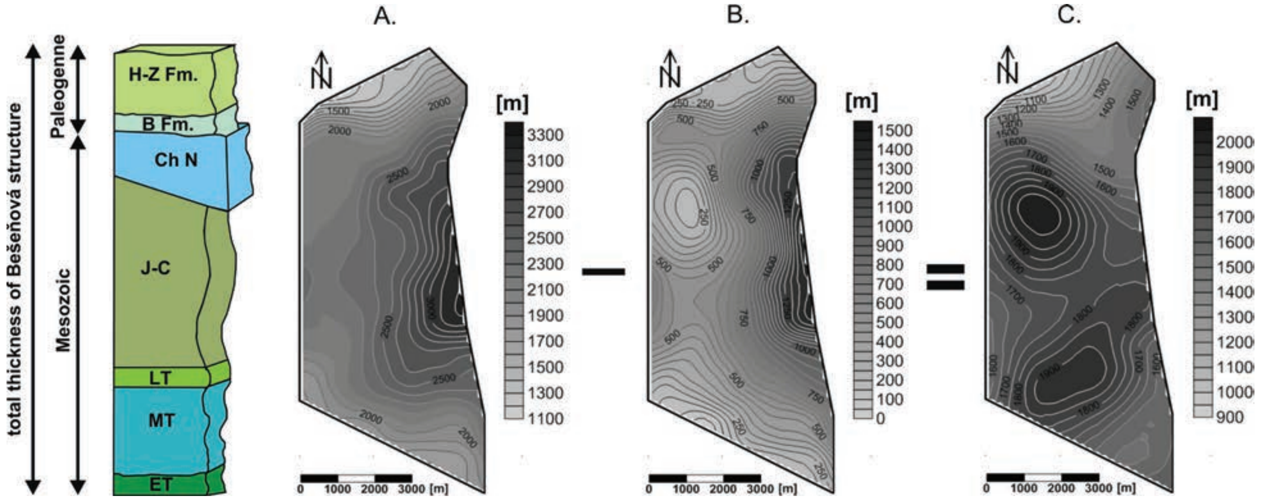


**Fig. 5. Step 2:** The total thickness map of Bešeňová structure (A.), obtained in Step 1 and used as an auxiliary variable during collocated cokriging to obtain Paleogene thickness map (B.). The map of the Mesozoic thickness (C.) is derived simply as a difference between the total thickness and the Paleogene thickness maps.

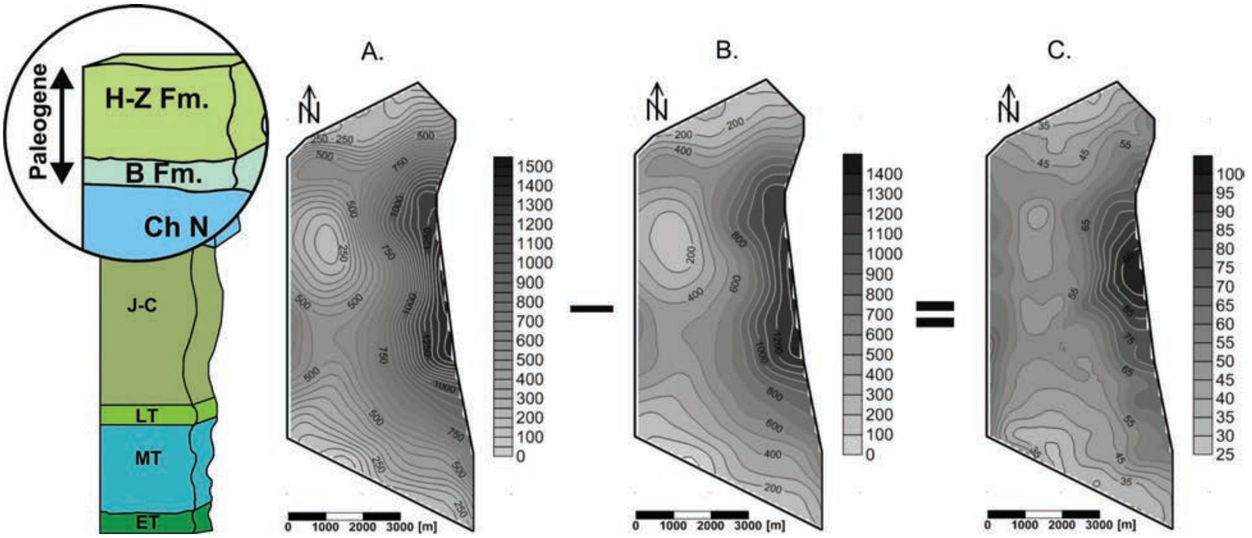


**Fig. 6. Step 3:** The map of Paleogene thickness (A.), obtained in Step 1 and used as an auxiliary variable during collocated cokriging to obtain H-Z Fm. thickness map (B.). The map of B Fm. thickness (C.) is derived simply as a difference between Paleogene thickness and H-Z Fm. thickness maps.

## Results

The Bešeňová structure can be considered as a faulted layer cake system of different chronostratigraphic units. It consists of seven units (zones) ranking from the Early Triassic to the Huty Formation of Paleogene. The aim of the study is to create three-dimensional model of the structure, which preserve the inherent spatial correlation between sequences of the units. Multivariate geostatistical modelling is based on the linear model of coregionalisation and uses a multivariate variogram model for estimation.

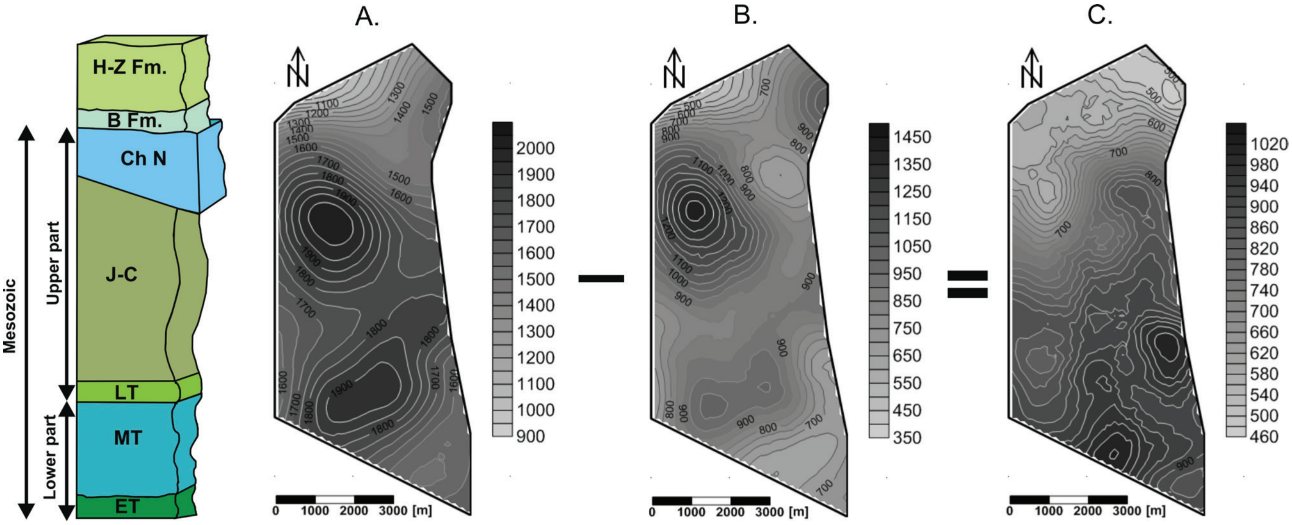


**Fig. 7. Step 4:** The map of Mesozoic thickness (A.), obtained in Step 2 (Fig. 2C.) and used as an auxiliary variable to cokrige the map of the UP (B.) using collocated cokriging. The map of the LP (C.) is derived simply as a difference between the total Mesozoic thickness and the UP thickness maps.

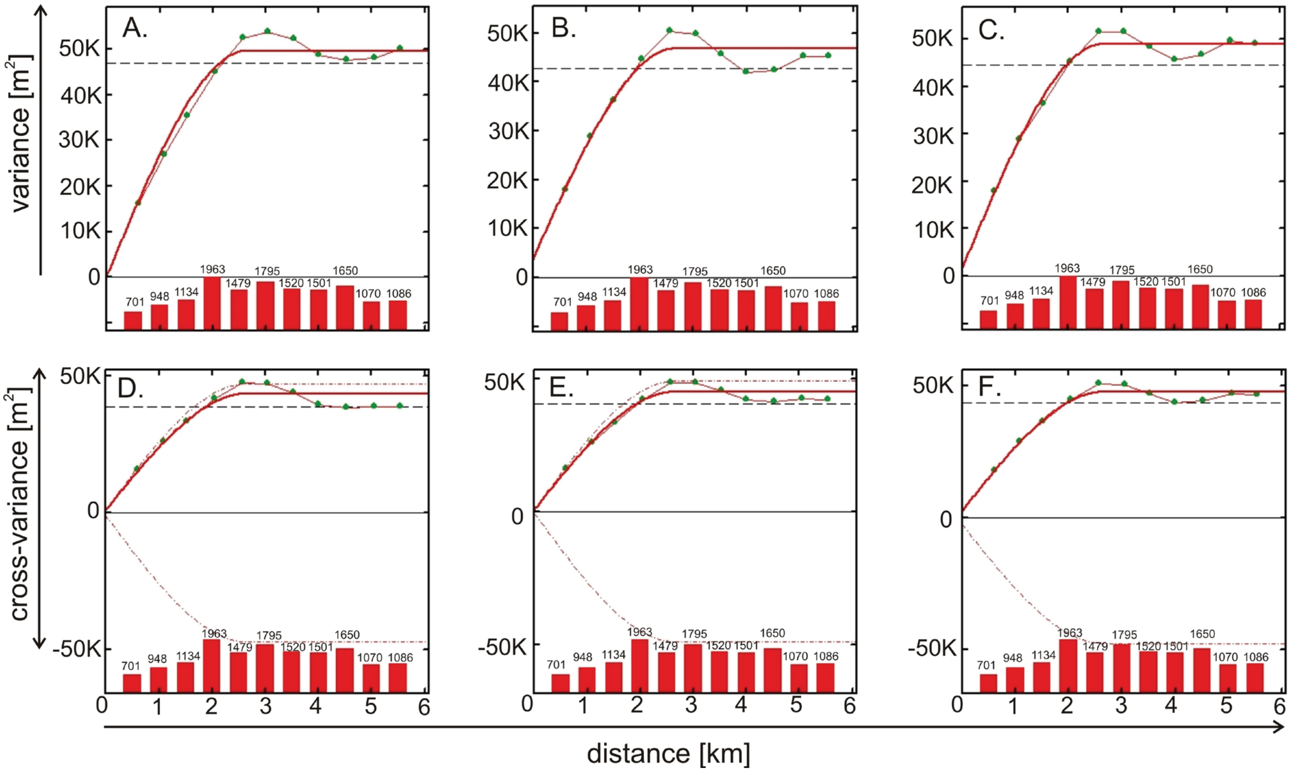


**Fig. 8.** The model of coregionalisation composed of three spatial variables with direct variograms: the Upper part of Mesozoic thickness, cumulative thickness of J-C and Ch N and J-C thickness (A., B., C.) and their respective cross-variograms (D., E., F.), showing very high positive correlation between pair of variables.

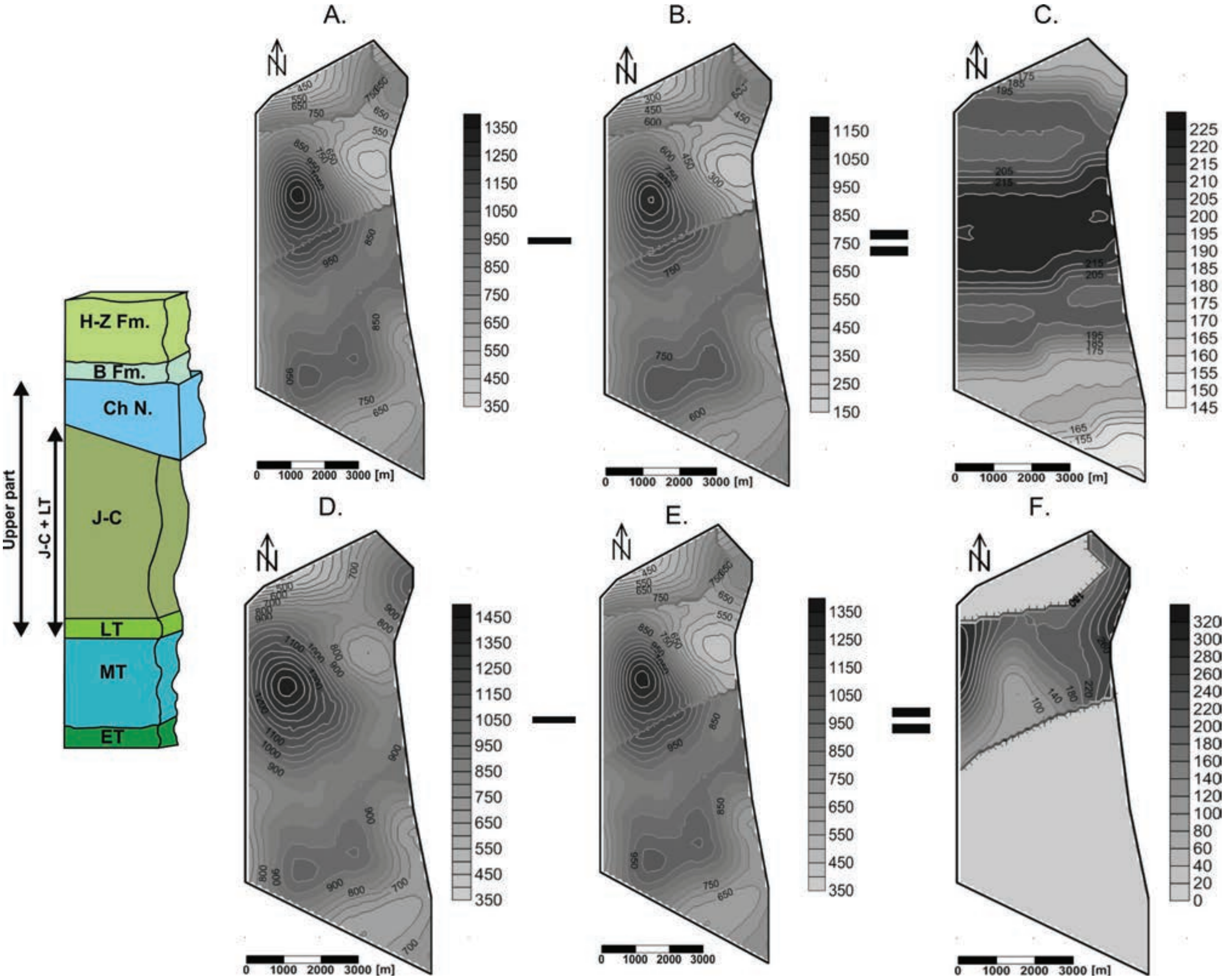


**Fig. 9. Step 5:** The cumulative thickness map of J-C and LT (A.) and thickness map of J-C (B.) are directly cokriged from the UP thickness map obtained in Step 4 (Fig. 4B.) and it is used as a collocated variable during cokriging estimation. The difference of A. and B. gives the LT thickness map (C.). Choc thickness map (F.) is obtained as a difference of the UP thickness map (D.), already known from Step 4, and the cumulative thickness map of J-C and LT (E.). Note that maps A. and E. are the same.

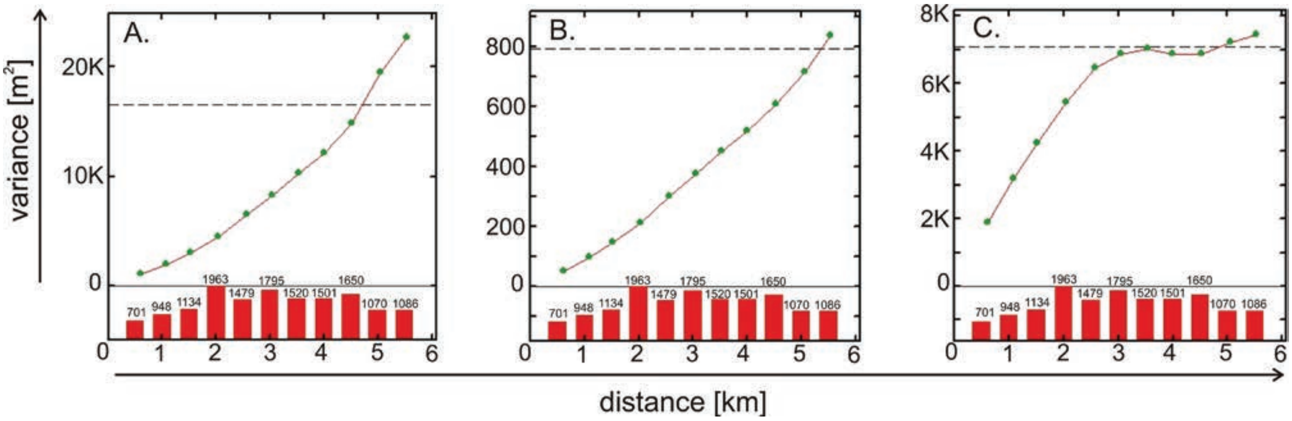


**Fig. 10.** Direct experimental variograms of MT and ET thicknesses (A., B.), showing strong increasing of variability with distance indicating non-stationary behaviour of the thickness variables. Because of stationary behaviour of the spatial variability of the LP (C.) it is not possible to create a permissible model of coregionalisation.

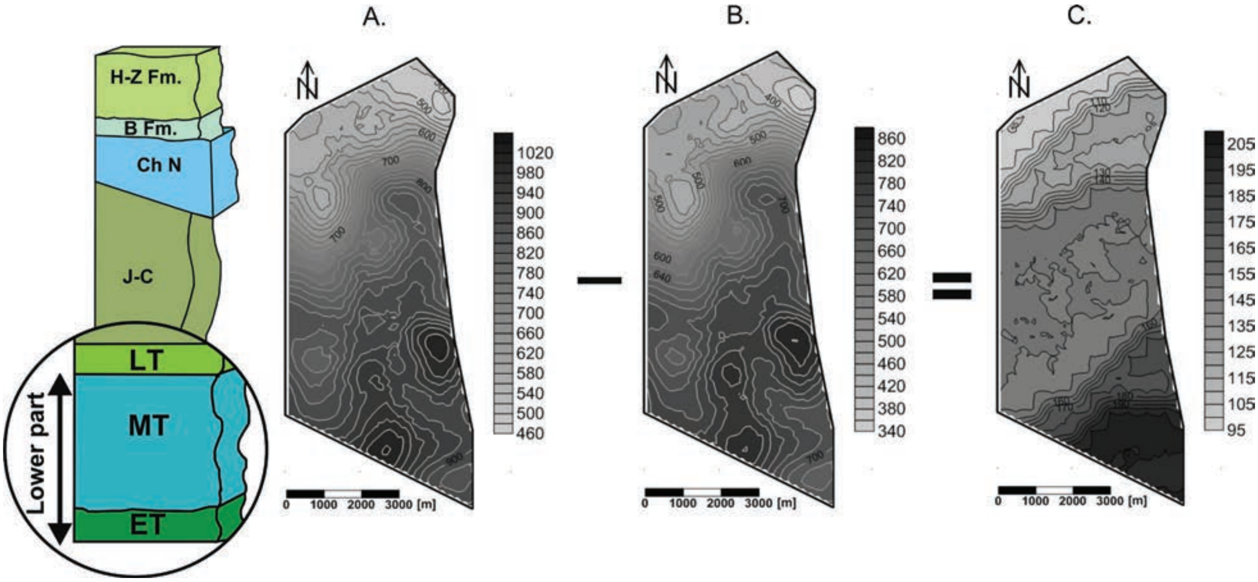


**Fig. 11. Step 6:** The map of Lower part of Mesozoic thickness (A.), obtained in Step 4 (Fig. 4C.) and used as an auxiliary external drift to krige the MT thickness map (B.). The ET thickness map is derived as a difference between A. and B.

As mentioned above, the full coregionalisation model calls for inferring $N(N+1)/2$ direct and cross variogram models. The difficulty lies in fact that these models cannot be modelled independently but mutually. That means increasing the number of regionalised variables $N$ makes the modelling of coregioanalisation (i.e. all direct and cross variogram) much more difficult because of the positive semi-definite condition of a permissible models of the basic structures (Goovaerts, 1997). For seven distinguished zones, plus total profile thickness, it calls for modelling of 36 direct and cross variograms. In practise, it is almost impossible due to different stationarity assumptions for different thicknesses. The cumulative thickness approach might be solution because it unifies the stationarity assumption, increases the correlations between the pairs of cumulative thicknesses and decreases the number of the direct and cross variograms but it is still 28 of them. Therefore, the modelling was performed in the following steps on the estimation grid with resolution 100 x 100 m in ISATIS environment:

1. In the first step, the total thickness (TT) of the Bešeňová structure between DEM and base of the structure was kriged using a direct model regionalisation (Fig 4A.) on the estimation grid with resolution 100 x 100 m. Because of lack of correlation between DEM and base of the structure, DEM was not used as auxiliary variable during modelling. The final TT map was used as a basic variable, vertically limiting the Bešeňová structure (Fig. 5A.).
2. In the next step, the Paleogene and Mesozoic thicknesses were modelled. Surprisingly, the Paleogene thickness had a higher correlation (0.78) with the TT (and better structured experimental cross-variogram as well) than the Mesozoic one (0.58) nevertheless three times higher proportion of the Mesozoic fulfilment of the Bešeňová structure. Due to this fact, the Paleogene thickness (Fig. 5B.) was cokriged using the model coregionalisation (Fig. 4) from the TT map (Fig 5A.), which was used as the collocated variable, already known in each location of the 100 x 100 m estimation grid. The Mesozoic thickness map was derived as a difference of the TT and the Paleogene thickness maps (Fig. 5C.).
3. Because of absence of a correlation between the B Fm. thickness and the Paleogene thickness, the H-Z Fm. thickness (Fig. 6B.) was cokriged from the Paleogene thickness map, obtained in **Step 2** (Fig. 5B.), using it as the collocated variable. To avoid the unacceptable results of the H-Z Fm. estimation in form of the negative thickness values where the H-Z Fm. absences, the input thickness data were transformed into Gaussian distribution with constrain the lowest back transformed thickness values. The B Fm. thickness map was derived as a difference of the Paleogene and H-Z Fm. thickness maps (Fig. 6C.).
4. The ET and MT thicknesses of the Mesozoic era show nonstationary behaviour with a trend of increasing of the thickness values from north to the south. They also show very high correlation with each other but no with the other horizons within the Mesozoic. From this reason, they were treated separately as a lower part of the Mesozoic (abbreviation LP). The cumulative thickness of the LT, J-C and the Ch N. constituted the upper part of the Mesozoic (abbreviation UP). Because of sta-

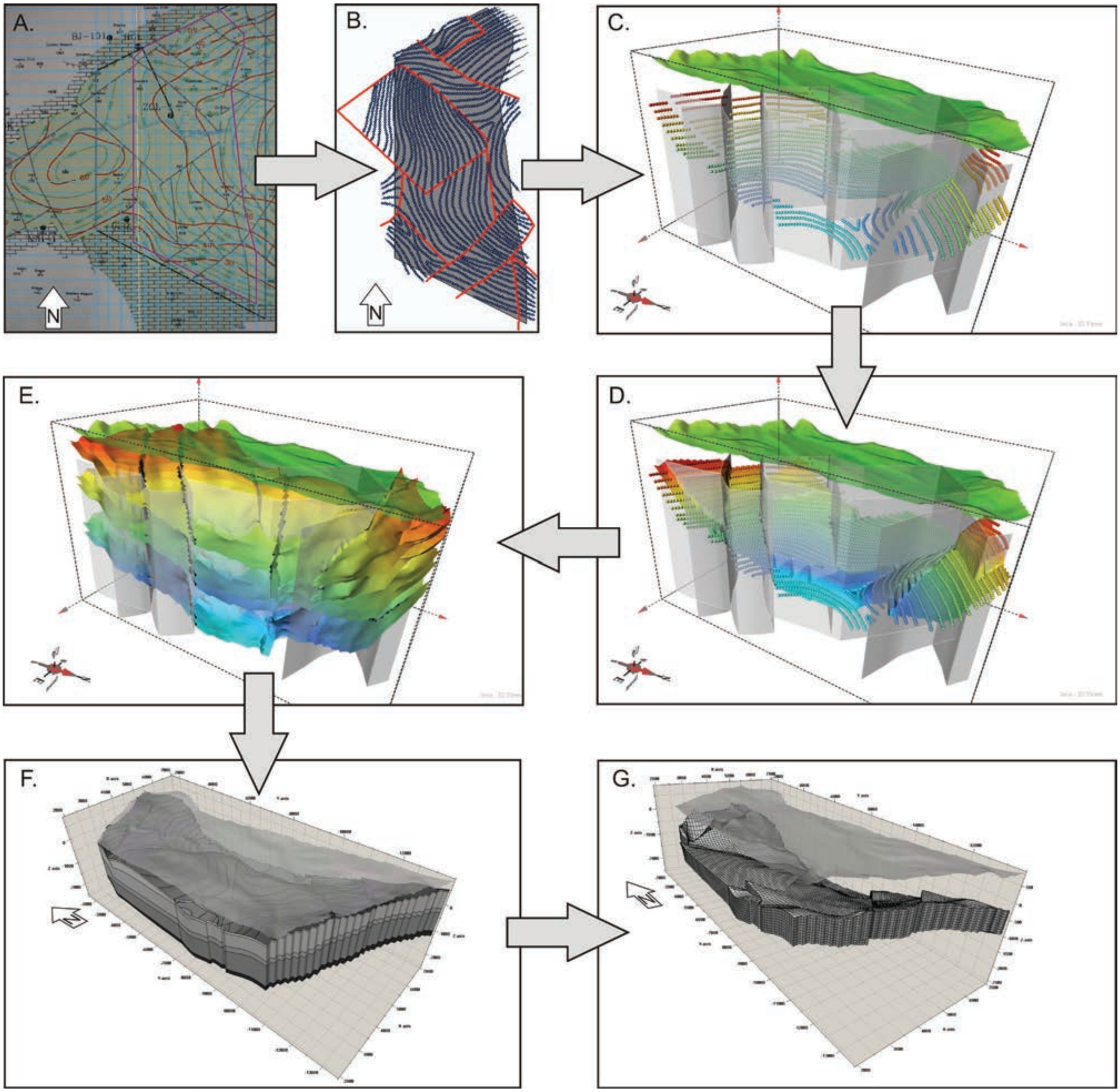


**Fig. 12.** Illustrative workflow of geometrical modelling of Bešeňová structure to obtain geometry and spatial position of hydrogeothermal reservoir: (A.) georeferenced hard copy of the structural maps and their digitalisation (B., C.) that are used to create the reference surfaces (D.) and using a model of the fault system (E.) are created systems of horizons of 3D Bešeňová structure (F.), from which a reservoir part of structure is delimited (G.).

tionary behaviour and strong correlation between the UP thickness and the total Mesozoic thickness, the UP thickness map was cokriged from Mesozoic thickness map (Fig. 7B.), obtained in **Step 2** (Fig. 5C.), which was used as the collocated variable. The difference between the Mesozoic and UP thickness maps resulted in the LP thickness map (Fig. 7C.).

5. The J-C thickness, which forms almost one third of the UP of the Mesozoic thickness, had very high correlation with UP thickness (0.86). The LT thickness of the UP of the Mesozoic thickness showed nonstationary behaviour with a trend of increasing values from the north and south part to the central one. The cumulative thickness of the LT and J-C showed stationary behaviour with well-structured variogram (Fig 8B.). From this, the multivariate coregionalisation model of three thickness variables – J-C, the cumulative thickness of J-C plus Late Triassic, and the total thickness of the UP of the Mesozoic – were modelled mutually, resulted in three direct variograms and three respective cross-va-

riograms (Fig. 8). The J-C thickness map (Fig. 9B.) and the cumulative thickness of the LT and J-C map (Fig. 9A.) were directly cokriged from the UP thickness map, previously obtained in **Step 4** (Fig. 7B.), as a collocated variable. The LT thickness map (Fig. 9C.) was obtained as a difference between the cumulative LT and J-C thickness map and cokriged J-C thickness map. The Ch N. thickness map (Fig. 9F.) was obtained as a difference between the UP thickness map and cokriged cumulative thickness of the Late Triassic and J-C map. Before it, the null thickness values of the Choč Nappe, indicating the absence of the Middle Triassic carbonates, were eliminate and the rest of values was delimitated by the respective polygon.

6. As pointed in **Step 4**, the ET and MT thicknesses both showed nonstationary behaviour of the spatial variability not reaching a sill (Fig. 10A., B.). Due to the fact, it is impossible to create a permissible model of coregionalisation with the LP of the Mesozoic thickness variable. The MT thickness forms the higher proportion of the LP thickness with very high correlation (0.98). First, the linear trend was extracted from the MT thickness values using the UP thickness variable as an external drift. Obtained residuals of the MT thickness were modelled as a stationary random function. It resulted in a variogram model used in universal kriging (Wackernagel, 2003) of the MT thickness using the LP thickness map, obtained in **Step 4** (Fig. 7B.), as an auxiliary external drift. Finally, the LT thickness estimation map was obtained by subtracting the MT thickness map from the LP one (Fig. 11C.).

Interpreted depth map of Middle Triassic base was digitized in ArcGIS environment and the structural surface was created (Fig. 12A. – D.). Using the thicknesses maps the rest of structural maps were derived. Created structural grids and DEM were imported into PETREL environment. Using the Early Triassic and Huty Fm. bases, a fault system was digitized and used to derive the final horizons from the structural surfaces (Fig. 12E., F.). Finally, using the fault system and the horizons, a new 3D grid was created for reservoir part of Bešeňová structure (Fig. 12G.).

## Discussion

Table 2 gives basic statistical characteristics of the modelled zone thicknesses obtained by multivariate global approach. The letter "E" in the table is for "Estimation" and it indicates the zone thickness cokriged using coregionalisation model from respective cumulative thickness. The letter "D" indicates the zone thickness obtained by subtracting from the respective cumulative thickness.

We focus at the total thickness in detail, while only summary statistics will be given for the other thicknesses. As mentioned previously, the TT was obtained by direct kriging from available thickness data without any additional auxiliary variables. In addition, as introduced before, the total structure thickness is critical for modelling process. It serves as an auxiliary variable to control the condition that the sum of the individual zone thicknesses will be equal to the total structure thickness. The range of estimated values of the TT is higher than the input data one as well as the variability of values. It is due to extrapolation of the high values in E and low values in W and NW part of studied area close to the domain boundaries as an effect of influence of a local spatial trend in the TT data within the estimation neighbourhood. The mean value is very well reproduced and very close to the experimental one.

**Tab. 2**

Basic statistical parameters of the thickness estimations for each zones obtained by global approach.

| Variable | | Min [m] | Max [m] | Mean [m] | SD* [m] |
|---|---|---|---|---|---|
| H-Z Fm. | E | 0.00 | 1 375.09 | 524.41 | 312.92 |
| B Fm. | D | 20.95 | 101.43 | 53.49 | 16.03 |
| Ch N. | D | 0.00<br>(100.00) | 300.00<br>(300.00) | 62.26<br>(192.06) | 96.75<br>(62.81) |
| J-C | E | 114.79 | 1 127.70 | 616.23 | 159.63 |
| LT | D | 141.93 | 231.36 | 194.95 | 20.30 |
| MT | E | 329.72 | 837.68 | 653.64 | 125.3 |
| ET | D | 98.45 | 201.41 | 149.37 | 27.13 |
| TT | E | 1 105.74 | 3 235.76 | 2 254.46 | 404.18 |

*SD – standard deviation
( ) – excluded zero values
E – estimated
D – derived

In general, we can observe a slight underestimation of the thickness range for the estimated zones. The minimal thickness value of H-Z Fm. is exactly 0 m as a result using non-linear estimation approach (Gaussian transformation). The maximum is slightly underestimated, which is balanced by derived B Fm. within the Pg. unit, which was used as the auxiliary variable in cokriging. Slightly worse situation appears for J-C zone with underestimation of all range of thickness values and reduced variability due to smoothing effect of kriging. Similarly, this range underestimation is balanced by derived LT thickness zones within the respective cumulative thickness. The opposite situation occurs in case of Mz zone where we can see overestimation of the range because of using non-stationary kriging with external drift of the lower part of Mz unit.

We compared the obtained results of the global approach with the traditional sequential modelling based on:

1. direct estimations of each individual zone thickness per-partes (DTE) or
2. direct estimations of base surfaces for each zone (DSE) and subsequent derivation of respective thickness as a difference of two consecutive surfaces.

Table 3 gives basic statistical characteristics of DTE approach where we can observe a systematic overestimation of range of values with decreasing their variability. Table 4 gives basic statistical characteristics of DSE approach. The negative minimal value signifies the presence of some negative values of H-Z Fm. thickness where base of the

**Tab. 3**

Basic statistical parameters of the thickness estimations for each zones obtained by DTE approach.

| **Variable** | **Min [m]** | **Max [m]** | **Mean [m]** | **SD* [m]** |
|---|---|---|---|---|
| H-Z Fm. | 0.00 | 1 390.00 | 523.37 | 320.76 |
| B Fm. | 28.66 | 101.09 | 53.71 | 14.54 |
| Ch N. | 87.11 | 314.04 | 183.32 | 41.20 |
| J-C | 75.30 | 1 141.14 | 615.29 | 188.67 |
| LT | 148.62 | 221.14 | 195.00 | 20.79 |
| MT | 389.03 | 808.13 | 651.98 | 126.11 |
| ET | 98.73 | 201.43 | 149.33 | 27.06 |
| TT | 1 078.55 | 3 209.58 | 2 249.52 | 406.71 |

*SD – standard deviation

**Tab. 4**

Basic statistical parameters of the thickness estimations for each zones obtained by DSE approach.

| Variable | **Min [m]** | **Max [m]** | **Mean [m]** | **SD* [m]** |
|---|---|---|---|---|
| H-Z Fm. | –25.67 | 1 380.30 | 511.95 | 319.21 |
| B Fm. | 28.29 | 100.77 | 53.67 | 14.60 |
| Ch N. | 40.18 | 317.33 | 187.98 | 67.82 |
| J-C | 75.51 | 1 147.18 | 620.83 | 190.44 |
| LT | 148.43 | 221.22 | 195.02 | 20.85 |
| MT | 387.75 | 807.23 | 652.09 | 126.29 |
| ET | 98.72 | 201.40 | 149.34 | 27.10 |
| TT | 1 028.72 | 3 364.36 | 2 243.91 | 413.24 |

*SD – standard deviation

zone intersect DEM. As in case of DTE approach, there are also systematic overestimation of range of values with decreasing their variability.

The performance of each approach, global, DTE and DSE, for TT estimation is evaluated by cross-validation. A TT experimental value is temporary removed from the data set. Then, TT value is re-estimated at the removed TT location from the remaining TT data within modelled estimation neighbourhood. The process is repeated for all TT data location. These estimates are compared with the true measured TT values to identify which approach performs best. Figure 13 shows three correlation graphs of TT values at data locations: A. global approach, B. DTE approach and C. DSE approach. The horizontal axes are for true TT values and the vertical axes are for the estimated values by each approach. Figure 13D. shows respective box plot of differences between true and estimated values of TT (estimation errors). It can be observed that the differences between estimates obtained by global approach and true values are the smallest with coefficient of correlation very close to 1 as well as the slope of linear regression equal to 1 indicating its closeness to

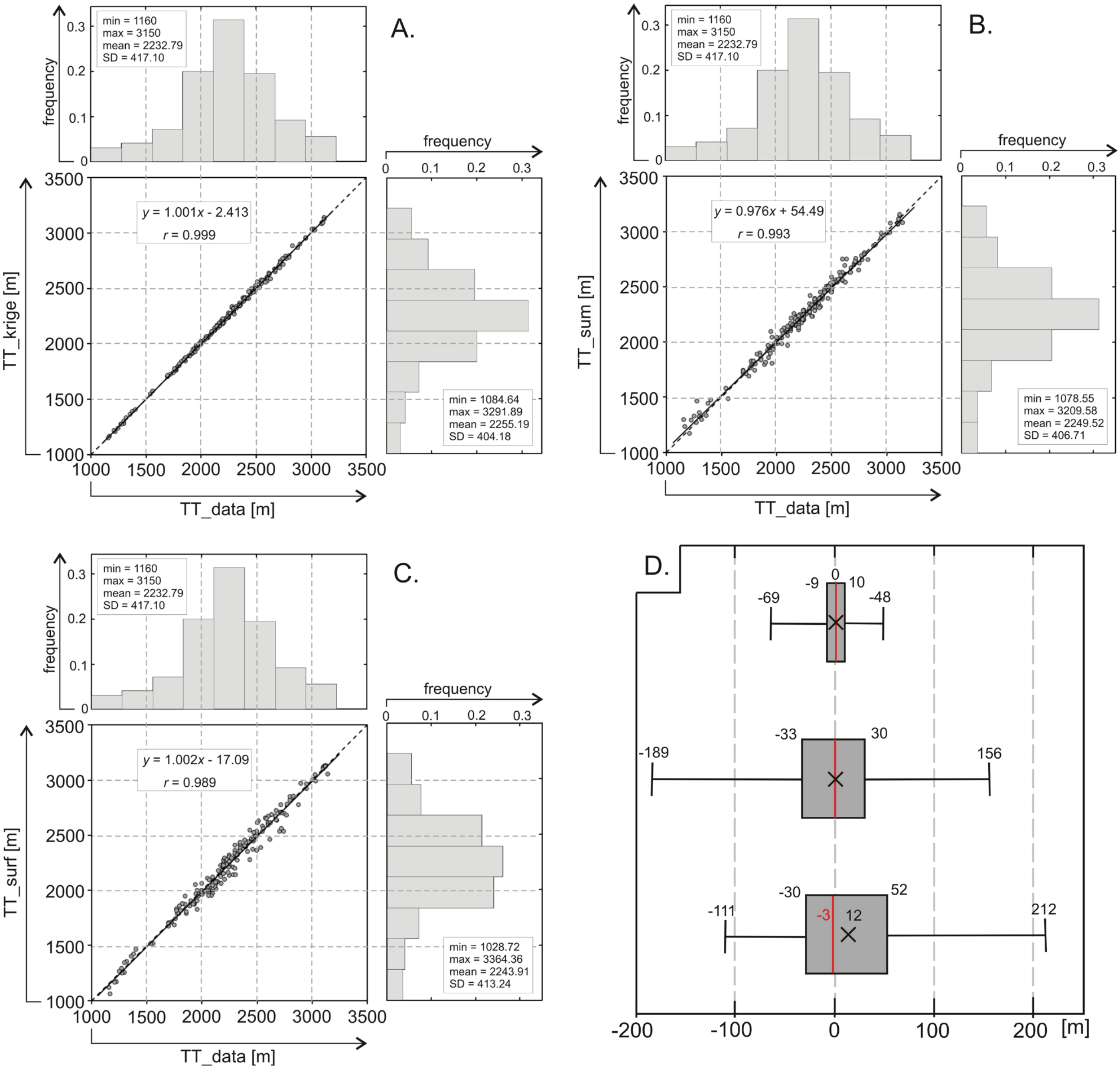


**Fig. 13.** Cross-validation scatterograms comparing the measured total thickness of Bešeňová structure versus the estimated ones using global approach (A.), DTE approach (B.) and DSE approach (C.). Graph D. shows the respective boxplots of differences between measured and estimated values of the total thickness.

the first bisector of the graph. Nevertheless, the kriging is exact interpolation, the negligible discrepancies between true and estimate values are caused by fact that the data locations do not coincide with the estimation grid nodes. The mean value of estimation errors is 0 m (unbiased estimation) with standard deviation less than 16 m.

It can be observed that for sequential approaches the discrepancies between true and estimated values become more visible and the correlations between true and estimated values become lower than 1. The DTE errors are perfectly balanced with mean value close to 0 m (unbiased estimation) but with much higher variance of errors than for global approach. The worst result for TT gives DSE approach, regardless producing the negative thickness values for the H-Z Fm., what calls for an artificial correction of these negative thicknesses.

The respective box plot suggest non-Gaussian distribution with higher mean value than the median and with higher proportion of the negative errors. That indicates mainly overestimation of the total thickness data.

The TT of the Bešeňová structure for three different approaches are shown in Fig. 14. In fact, the map of thickness distribution shown in Fig 14A. is the same as in Fig. 5A., obtained by global approach estimation. The resulting map of the TT obtained by the DTE approach (Fig. 14B.) shows an artificial increasing of the structure thickness.

This unacceptable artefact coincides with the Ch N. zone and it appears as a result of the individual modelling of zone thicknesses and successive adding to each other to build the complete structure model instead of mutual modelling of the thicknesses to keep a geometrical correlation among them. The resulting map of the TT obtained by the DSE approach (Fig. 14C.) is very similar to the one obtained by global approach but, as commented previously, it gave the worst cross-validation score and produces the negative thickness for H-Z Fm.

Since the global approach estimation gives the best cross-validation score, we compared the results of sequential DTE and DSE approaches with it. The comparison is showed in Fig. 14D. as the box-plots. The differences between TT from global approach versus DTE and DSE respectively are very similar to these obtained by the cross-validation procedure.

## Conclusion

The paper demonstrate the application multivariage geostatistics to build 3D geological model of the hydrothermal subsurface structure Bešeňová. It is compared to the traditional univariate approaches based on direct modelling of the individual zone thicknesses or direct modelling of internal surfaces. The first traditional approach led to the artificial increasing of the total structure thickness due to presence of the incomplete Choc nape within the structure. Second approach gave some negative thickness values for the uppermost H-Z formation that resulted in crossing the formation base horizon with the DEM.

The global approach is based on the multivariate modelling of the cumulative thickness of the zones within

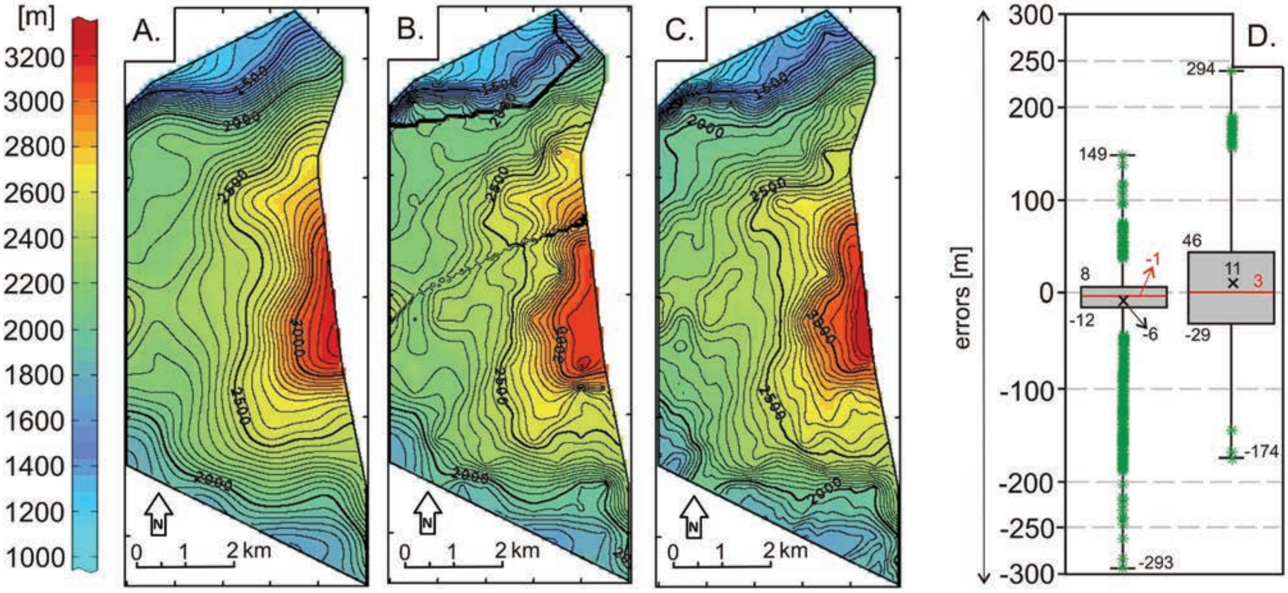


**Fig. 14.** Three maps of the total thickness of Bešeňová structure obtained by different approaches: A. using direct geostatistical modelling, B. using DTE approach based on summation of all zone thicknesses modelled individually, C. using DSE approach based on summation of all zone thicknesses derived from base surfaces modelled individually.

the total formation thickness. The global approach has proved to be very useful one that produced a geometrically reliable geological model of the subsurface structure with maintaining the consistency between the zones and the total thickness of the formation. The multivariate methods, described and applied in the paper, provide an efficient way of estimating the isochore maps and deriving the internal horizons to build a subsurface structure model.

Presented 3D geological model has already found its use in multiple aspects of geothermal research of the Bešeňová elevation hydrogeothermal structure. Geometry of the model, i.e. true depth of horizons and overall thickness of each unit has been used in reconstruction of stationary geothermal model and onward analysis of reservoir dynamics; such is occurrence of separated convection cells formed under various rate of reservoir base overheating (Fričovský et al., 2014b). Reservoir volumetrics were used to assess geothermal resources and reserves base for both, the deep and shallow geothermal reservoir, including sustainable production capacity, and recovery rate (Fričovský et al., 2014a, c). A model complexity, including structural dissection of the deep and shallow reservoir body, faulting, lithology and structural geometry provided critical background for conceptual site modelling with robust use of mixing and boiling models (Fričovský et al., 2015), hydrochemical facial analysis, and a complete scale of geothermometry (Fričovský et al., 2016), ranging from solute to a multicomponent.

However, there is a rising call to prompt research on sustainable geothermal energy use in the country. The Bešeňová elevation hosts one of most popular thermal parks and individual space heatings in Slovakia, turning the site among most important. Presence of geothermal resources in underlain Tatricum Envelope Unit (Mid Triassic carbonates) is – at the best – questionable, especially when concerning conventional "wet" geothermal reservoirs (e.g. Fendek et al., 1988; Remšík et al., 1998; Fendek et al., 2005). This gives a model a sufficient scale for numerical flow models and site reservoir management, probabilistic resource assessment or involves a role in future studies on hydraulic and/or thermal communication between the Bešeňová elevation and the Lúčky – Kalameny structure.

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# Použitie metód viacpremennej geoštatistiky na vytvorenie geologického modelu hydrogeotermálnej štruktúry Bešeňová

Výskumu geotermálnych rezervoárov v zmysle rezervoárového inžinierstva, ktorého náplňou je okrem iného aj modelovanie dynamického toku, odpoveď rezervoáru, hodnotenie zdrojov alebo nastavenie produkcie geotermálnej energie či reinjektáže, je nevyhnutný 3D geologický model. Hlavný dôraz pri geologickom modelovaní podpovrchovej štruktúry sa kladie na správnu priestorovú geometriu a postupnosť rôznych chronostratigrafických alebo litostratigrafických jednotiek na správny odhad a simuláciu priestorového rozloženia teplôt a ďalších študovaných parametrov a premenných štruktúry používaných na hodnotenie geotermálnych rezervoárov. Cieľom prezentovaného článku je vytvoriť 3D geologický model hydrogeotermálnej elevačnej štruktúry Bešeňová s použitím viacrozmerného prístupu na zachovanie skrytej geometrickej korelácie medzi siedmimi geologickými jednotkami bez použitia umelých korekcií, ako je eliminácia zápornej hrúbky v niektorých oblastiach alebo korekcia celkového objemu štruktúry. Článok tiež porovnáva výsledky tradičných sekvenčných prístupov založených na jednorozmernom modelovaní jednotlivých hrúbok v prípade každej zóny alebo príslušných základných horizontov.

Motiváciou priestorového modelovania podpovrchových štruktúr je modelovanie povrchov a odvodenie jednotlivých horizontov tvoriacich študovanú štruktúru. Cieľom priestorového modelovania je vytvorenie priestorového modelu vnútornej stavby podpovrchovej štruktúry pri zachovaní inherentnej geometrickej korelácie. Hlavný dôraz pri modelovaní priebehu horizontov podpovrchových štruktúr sa kladie na zachovanie skrytej geometrickej korelácie medzi geologickými jednotkami, ktorá je založená na dvoch dôležitých predpokladoch:

1. Hrúbka akejkoľvek zóny v rámci určitej formácie musí byť v každom bode študovaného priestoru rovná alebo väčšia ako nula. To znamená, že výsledná mapa izochor danej zóny musí byť striktne kladná priestorová premenná.
2. Suma hrúbok všetkých zón v rámci určitej formácie musí byť rovná celkovej hrúbke tejto formácie.

Metódy modelovania vnútorných horizontov geologických objektov podpovrchových štruktúr sa rozdeľujú na dve hlavné skupiny. Prvou sú tradičné sekvenčné metódy. Tieto metódy modelovania vnútorných horizontov sú založené na samostatnom modelovaní buď priebehu jednotlivých vnútorných horizontov v rámci študovanej formácie, alebo hrúbky jednotlivých zón v rámci študovanej formácie. V prvom prípade je modelovanie jednoznačné a založené priamo na hĺbkových údajoch. Samostatné modelovanie priebehu jednotlivých vnútorných povrchov však nezabezpečuje prvý predpoklad. Odvodením hrúbky jednotlivých zón v rámci formácie na základe takto získaných horizontov často vedie k neželanej zápornej hrúbke niektorých zón z dôvodu pretínania sa jednotlivých horizontov. V prípade priameho odděleného modelovania hrúbky jednotlivých vnútorných zón sa modeluje hrúbka zón postupne, jednotlivo pre každú zónu samostatne, spravidla od určitého referenčného povrchu, napr. stropu formácie. Na základe týchto hrúbok v podobe máp izochor je odvodený priebeh jednotlivých vnútorných horizontov vymedzujúcich zóny. Vážnym nedostatkom je, že suma hrúbok jednotlivých vnútorných zón nesúhlasí s celkovou hrúbkou formácie, čím dostávame kladnú alebo zápornú chybu odhadu. Takáto situácia sa rieši na základe proporčnej alebo rovnomernej korekcie týchto chýb, tzv. korekcie objemu formácie.

Cieľom článku je aplikácia tzv. globálnej metódy odhadu vnútorných horizontov, založenej na nepriamom prístupe viacpremenného modelovania kumulatívnych hrúbok jednotlivých zón študovanej formácie a celkovej hrúbky formácie. Globálne modelovanie kumulatívnych hrúbok zón v rámci formácie nie je priamočiary proces ako v prípade odděleného modelovania hrúbky príslušných zón. Okrem toho, že je eliminovaný problém nekonzistencie celkovej skutočnej hrúbky formácie s modelovaným, kumulované hrúbky na rozdiel od jednotlivých hrúbok zón sú dobre priestorovo korelované a vo vzájomnom vzťahu. To uľahčuje tvorbu lineárneho modelu koregionalizácie. V prípade modelovania kumulatívnych hrúbok sa eliminuje aj problém vyplývajúci z rozdielnosti predpokladov stacionarity hrúbky jednotlivých zón v rámci študovanej formácie, keď nie je ani teoreticky možné vytvoriť lineárny model koregionalizácie medzi takýmito premennými. Najväčším problémom aplikácie globálnych metód je ich pomerne zložité matematické pozadie, ako aj ich dostupnosť v rámci softvérových riešení. Napriek tomu je však čas vynaložený na aplikáciu globálnych metód prínosom v prospech kvality a konzistencie vytvoreného geologického modelu bez potreby umelého zásahu a korekcie do získaných výsledkov. Tento článok preukázal výhody a kvality globálnych metód modelovania priebehu vnútorných horizontov podpovrchových štruktúr v porovnaní s tradičnými sekvenčnými metódami.

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